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Rotating Machinery Diagnosis Using Wavelet Packets-Fractal Technology and Neural Networks

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Abstract

This paper presents a new fault diagnosis procedure for rotating machinery using the wavelet packets-fractal technology and a radial basis function neural network. The main purpose is to investigate different fault conditions for rotating machinery, such as imbalance, misalignment, base looseness and combination of imbalance and misalignment. In this study, we measured the non-stationary vibration signals induced by these fault conditions. Applying wavelet packets transform to these signals, the fractal dimension of each frequency channel was extracted and the box counting dimension was used to depict the failure characteristics of the fault conditions. The failure modes were then identified by a radial basis function neural network. An experiment was conducted and the results showed that the proposed method can detect and recognize different kinds of fault conditions. Therefore, it is concluded that the combination of wavelet packets-fractal technology and neural networks can provide an effective method to diagnose fault conditions of rotating machinery.

Keywords: Fault diagnosis; Rotating machinery; Wavelet packets; Fractal; Box counting dimension; Radial basis function neural network

1. Introduction

Rotating machinery faults detection and diagnosis is typically based on the measurements of vibration (Lyon, 1998; Tandon and Choudhury, 1999). The spectral content of the emitted vibration signals are analysed to ascertain the current operating condition of the machinery. Conventionally, the faults of rotating machinery are usually diagnosed by the Fast Fourier Transform (FFT) to obtain the power spectrum of raw signals. Although the Fast Fourier Transform is an effective method and is widely used in signal processing and machinery diagnosis, but it is only suitable to stationary signal processing and the transformed signal loses its raw information of time domain completely. The limitation of the Fast Fourier Transform in analyzing non-stationary signals leads to the introduction of time-frequency or time-scale signal processing methods. Recently, the application of Wavelet packets transform is more widely used to analyze non-stationary signals, and has been proven effective in fault diagnosis (Boulahbal and Golnaraghi, 1999; Geng and Qu, 1994; Lu et al., 2006; Liu, 2005; Zhang et al., 2003). Wavelet packets transform is a multi-resolution time-frequency method and highlights the localized signal characteristics in time and frequency domains (Mallat, 1998; Wang, 2001). Thus, in processing, signals can be presented from general to specific different scales. But other key issues in faults diagnosis are how to extract the

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effective characteristics from the processed signal and how to describe the non-stationary fault conditions in a quantitative way.

Fractal refers to the fact that geometric dimension of some objects is a fraction rather than an integer (McCauley, 1993). It is able to describe geometric objects that are not possible in traditional Euclidean geometry. Fractal was firstly introduced by Mandelbrot in his study of coastlines. Fractal analysis has now become widely used because of its universal existence (Mandelbrot, 1977; 1983), including philosophy, economics, information, chemistry (Huang and Shi, 2004), medical science, and geography. Academics use fractal theory to interpret variable and unpredictable physical phenomenon, and have obtained substantial results in image and signal processing (Popescu et al., 1997). But the application of fractal analysis to fault diagnosis in rotating machinery is still in its early stages.

Neural networks are widely applied in the field of pattern recognition and classification because of their capacity to map non-linear functions (Arbib, 2002). Many types of networks have been successfully developed for faults classification of automatic machines. For example, Back Propagation (BP), Adaptive Resonance Theory (ART), Bidirectional Associative Memory (BAM), Brain State in a Box (BSB), Radial Basic Function (RBF) (Catelani and Fort, 2000; Chen et al., 1991; Lai et al., 2004). Among these, the back-propagation network (BP), a multilayer feed forward network, is the most popular one used in engineering applications. Many improved algorithms have been developed for BP networks to increase the speed of training and to avoid falling into local minimum during training. But their effectiveness in solving these problems are still not good. Furthermore, the determination of a suitable architecture for BP networks is difficult for inexperienced users, especially in terms of the number of neurons in the hidden layer (Fan et al., 2002; Haykin, 1999). The radial basis function (RBF) network is a new and more effective training method. It avoids complicated calculations and the time required for training is shorter than for that of BP networks. In addition, it is capable of fast convergence and automatically determining the number of neurons in the hidden layer during training. Hence, in general, an optimum can be obtained by using a RBF network.

Faults diagnosis of rotating machinery refers to the evaluation of whether the machinery equipment is running normally, based on the vibration signals generated during the operation, and identification of the cause and portion of the faults. In this study, the vibration signals induced by the imbalance, misalignment, base looseness and combination of imbalance and misalignment are measured. The wavelet packets analysis is used to decompose the measured signals into a series of localized wavelet functions. Then, fractal theory is used to calculate the box-counting dimension of different frequency sections, and a structure fractal dimensions character vector is then established in a quantitative manner. Based on this vector, a radial neural network is applied to develop a fault model. An experiment was conducted to evaluate the performance of the new fault diagnosis procedure.

2. Theory

2.1 Wavelet packets transform

Wavelet packets are a collection of functions $\left\{2^{-j/2}\mu_n(2^{-j}t-k)\right\}$ $(n\in N, j,k\in Z)$ which are generated from the following sequence of functions:

$$\begin{cases}\mu_{2n}(t)=\sqrt{2}\sum_{k\in Z}h(k)\mu_n(2t-k)\\ \mu_{2n+1}(t)=\sqrt{2}\sum_{k\in Z}g(k)\mu_n(2t-k)\end{cases}\quad n\in Z \tag{1}$$

Where $h(k)$ and $g(k)$ are the Quadrature Mirror Digital Filters (QMF) associated with the scaling and wavelet functions, as shown below:

$$\begin{aligned}h(k)&=\frac{1}{\sqrt{2}}\left\langle\phi\left[\frac{t}{2}\right],\phi\left[t-k\right]\right\rangle\\ g(k)&=\frac{1}{\sqrt{2}}\left\langle\varphi\left[\frac{t}{2}\right],\phi\left[t-k\right]\right\rangle\end{aligned}\quad t\in R, k\in Z \tag{2}$$

As $n=0$, Eq. (1) can be rewritten as:

$$\begin{cases}\mu_0(t)=\sqrt{2}\sum_{k\in Z}h(k)\mu_0(2t-k)\\ \mu_1(t)=\sqrt{2}\sum_{k\in Z}g(k)\mu_0(2t-k)\end{cases} \tag{3}$$

Where $\mu_0(t)$ and $\mu_1(t)$ correspond to the wavelet function $\varphi(k)$ and the scale function, $\phi(t)$ respectively, i.e. $\mu_0(t)=\varphi(t)$, $\mu_1(t)=\phi(t)$.

Because wavelet packets decomposition is ortho-

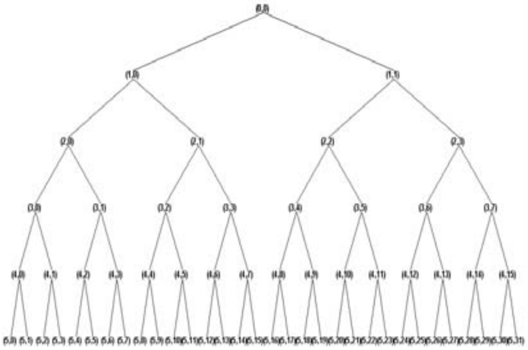

Fig. 1. Wavelet packets decomposition tree.

gonal, it retains the time domain characteristics of the raw signals in different dimensions. Therefore, in the process of faults diagnosis, when applying wavelet packets analysis, the vibration signals are decomposed and reconstructed by using the following equations:

$$\begin{cases} s_k^{2m} = \frac{1}{2}\sum_L s_L^m h_{L-2k} \\ s_k^{2m+1} = \frac{1}{2}\sum_L s_L^m g_{L-2k} \end{cases} \tag{4}$$

$$s_k^m = \sum_L s_L^m h_{L-2k} + \sum_L s_L^m g_{L-2k} \tag{5}$$

This paper will use wavelet packets analysis to decompose signals into independent frequency bands to extract characteristics of each frequency bands. When the signals are decomposed into N level, it will result in 2^N sub-bands of frequency. Wavelet packets decomposition forms a complete binary tree, as depicted in Fig. 1.

2.2 Fractal theory

Fractal Dimension is an important parameter in quantitatively describing fractal characteristics and is normally used to describe the complexity of signals. Put simply, different signals contain different fractal dimensions. Thus, fractal dimensions can extract the effective characteristics of signals. Fractal dimensions have developed into more than a dozen different dimensions, including Hausdorff, box counting, capacity, correlation, and Lyapunov dimensions. The relatively simple calculation of the box counting dimension is favored by many academics. This paper will use it to analyze the fractal phenomenon of different frequency bands of vibration signals generated by different fault conditions.

The algorithm of box counting dimensions assumes a time serial signal $s(j) \subset S$, where S is a close set R^n in n dimension Euclidean space. Dividing R^n into a very fine grid and letting N_r be the number of grids in a discrete space set of grid of *r*, then the box counting dimension is defined as:

$$d_B = \lim_{r \to 0}\left(-\frac{\log(N_r)}{\log(r)} \right) \tag{6}$$

Since the highest recognition rate of time series is $s(j)$ and the sampling interval of r is unable to approach zero according to its definition so in the actual calculation it will use an approximate algorithm and gradually enlarge grid r to grid k_r, where $k \in Z^+$. Set N_r as the number of grids in a discrete space S with a grid width of k_r, then N_r can be calculated by using the following formula:

$$\begin{aligned} P(kr) = \sum_{j=1}^{N_0/K} \Bigg| & \max\left\{ \begin{matrix} s_{k(j-1)+1}, s_{k(j-1)+2}, \dots \\ \dots, s_{k(j-1)+k+1} \end{matrix} \right\} \\ & - \min\left\{ \begin{matrix} s_{k(j-1)+1}, s_{k(j-1)+2}, \dots \\ \dots, s_{k(j-1)+k+1} \end{matrix} \right\} \Bigg| \\ & j = 1,2,\dots\dots, N_0/k \\ & k = 1,2,\dots K, K < N_0 \end{aligned} \tag{7}$$

$$N_r = P(kr)/(k_r) + 1 \quad N_r > 1 \tag{8}$$

The grid number N_r and the grid width k_r can produce a dual logarithm curve ($\log(N_r) - \log(k_r)$). In the curve, one can check for the existence of a Scale-Invariant region. Because, in general, the fractal phenomenon is not always present, but only demonstrates fractal characteristics within a certain scale. Assuming the beginning and end of the scale invariant region is k_1 and k_2, then the $\log(N_r) - \log(k_r)$ curve in this region should fulfill the following linear regression model:

$$\log(N_r) = -d_B \log(k_r) + b \tag{9}$$

One can apply the least square method to obtain the slope of the straight line and the box counting dimension is the slope of the straight line. The fractal size of the curve is determined by the range occupied by the curve in the space. For example, for a straight line, the size of the fractal dimension is equal to topology dimension 1.0. For example, Fig. 2 shows the $\log(N_r)-\log(k_r)$ curve of the linear function of $f(x)=2x+3$. The estimated value of box counting dimension is 1.048, and is consistent with the theoretical value.

2.3 Radial basis neural network

RBF Neural Network uses a Gaussian Function as its basis and comprises a forward network. The Gaussian function of the transfer function of the hidden layer as follow:

$$G_k = \exp\left(\frac{-\left|x-x_k\right|^2}{\sigma^2}\right) \quad (10)$$

Where x_k is the center of the kth neuron Gaussian function and σ is the unitary parameter and controls the radial function range of the function. The output of the RBF neural network radial function from input layer to hidden layer is a non-linear mapping and the output layer is linear. In other words, it transforms the primary non-linear divisible characteristics into another space (generally high dimension space). Through such a transformation, the raw problem is linearized in the new space and a linear element can be used to solve it.

The RBF neural network is a feed forward network and the architecture is shown in Fig. 3. The neural

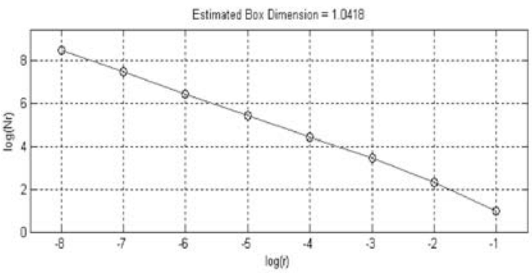

Fig. 2. Straight line box counting dimension estimation curve.

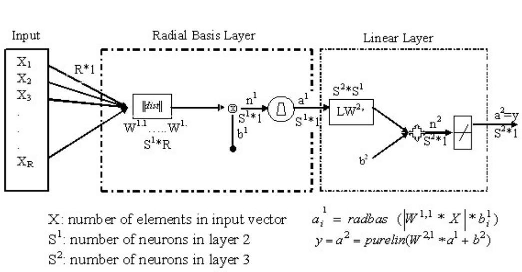

Fig. 3. Structure of RBF neural network.

network consists of three layers, i.e. the input layer with $R*1$ input neurons, a hidden radial basis layer of S^1*R neurons, and an output linear layer of S^2*1 neurons. In Fig. 3, $|dist|$ is the Euclidean distance between input weight $W^{1,1}$ and the input vector X.

The mathematical equation of hidden layer output a^1 is:

$$a^1 = \left|W^{1,1}-X\right| * b^1 \quad (11)$$

Where b^1 is the bias vector of the hidden layer.

Each neuron of the RBF neural network output is a value that depends on its weight from the center of the RBF neural network, which uses a Gaussian transfer function in the hidden layer and a linear function in the output layer. The output of the RBF neural network linear layer is given by:

$$y = a^2 = W^{2,1} * a^1 + b^2 \quad (12)$$

Where b^2 is the bias weight vector of the output layer.

This paper uses the RBF neural network to recognize the fault conditions of rotating machinery. The fault characteristics and cause of faults are the nodes of input layer and output layer, respectively, and conducted with network learning and training. Therefore, in the RBF neural network faults recognition model, the input vector X is the fault characteristics and the output vectors are the corresponding faults. Based on the learning and training of test samples of measured signals, the network structure can be adjusted to an effective recognition tool in the fault diagnosis of rotating machinery.

In summary, this paper presents a new fault diagnosis approach using wavelet fractal techniques and neural networks. The approach is illustrated in Fig. 4. When conducting a fault diagnosis of rotating machinery, the first step is to measure the vibration signals, perform wavelet packets decomposition, and categorize the results in each section of a perpendicular frequency band. The next step is to calculate the box counting dimension of the processed data in each section in order to generate a characteristic vector set, which provide the input for the RBF neural network. Based on the learning and training of network, the causes of faults can then be

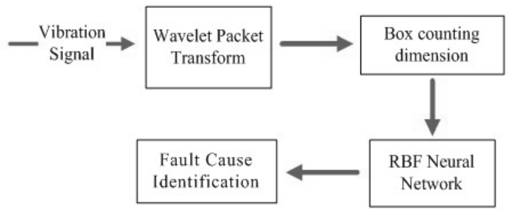

Fig. 4. The fault diagnosis procedure of rotating machinery.

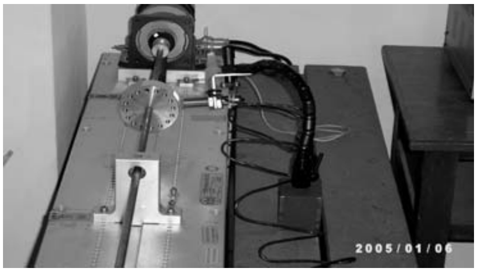

Fig. 5. Experimental set-up.

identified.

3. Experimental analysis and discussion

3.1 Experimental set-up

The experimental set-up is shown in Fig. 5. A small rotor kit is used to simulate a rotating machine. Five conditions are considered in this study, as follows:

1. Condition A: normal (good) condition;
2. Condition B: imbalance condition;
3. Condition C: misalignment condition;
4. Condition D: the base looseness condition; and
5. Condition E: combination of condition B and condition C.

An accelerometer (Bruel & Kjaer 4384) and amplifier (Bruel & Kjaer 2635) were used to measure the vibration signals. The time domain signals of different fault conditions are shown in Fig. 6. An inductive proximity probe was also used to collect rotation tachometer signals. The measured signals were recorded on magnetic tape and then digitized through a acquisition unit (National Instrument, Daq-1200), and analyzed using Matlab software. The analysis frequency range was set at 1000 Hz. The sampling frequency was 2560 Hz. The number of sampling data was 4096 points in all the investigated cases. The rotating speed of the rotor kit was set at 2400 rpm, i.e., the base frequency (f_b) was 40 Hz. Total of 20 groups of vibration signals were collected for each fault condition. In these groups of signals, 12 groups were used to train the RBF neural network. The rest 8 groups were used as test samples to evaluate the recognition ability of the neural network.

After extracting the vibration signals of different fault conditions, the raw signals were examined to see if they contained fractal phenomenon. A fractal logarithm curve, as shown in Fig. 7, was obtained by a box counting dimension calculation of the signals.

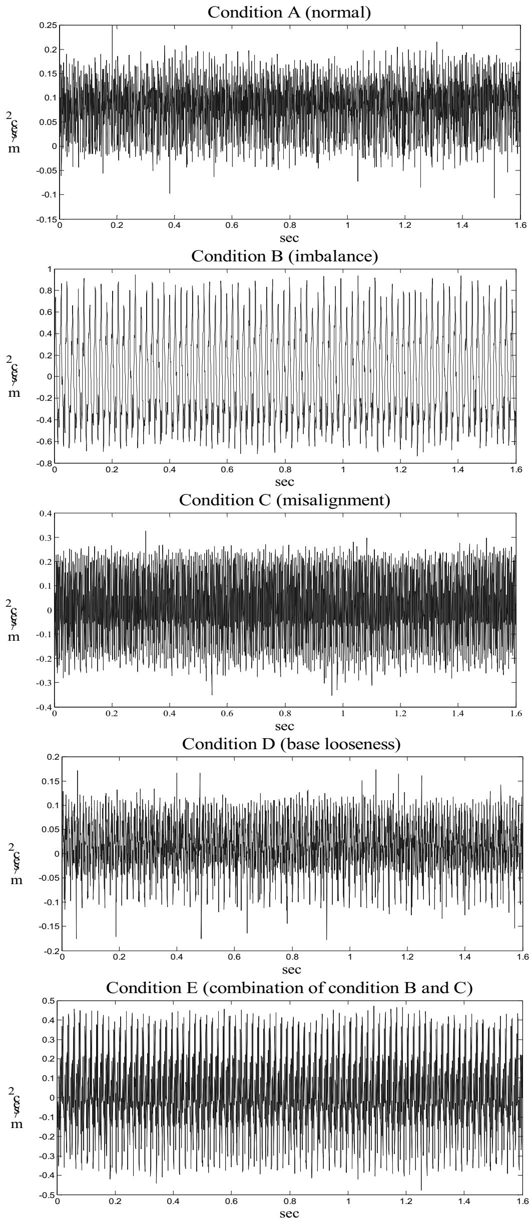

Fig. 6. Measured vibration signals in time domain.

The figure showed that logarithm curve has a linear line at the scale-invariant region. This phenomenon confirms the measured signals are suitable for the application of fractal analysis. The box counting dimensions estimation results of different fault conditions were indicated in Fig. 8. The maximum value of box counting dimensions is the condition E due to the combination effect.

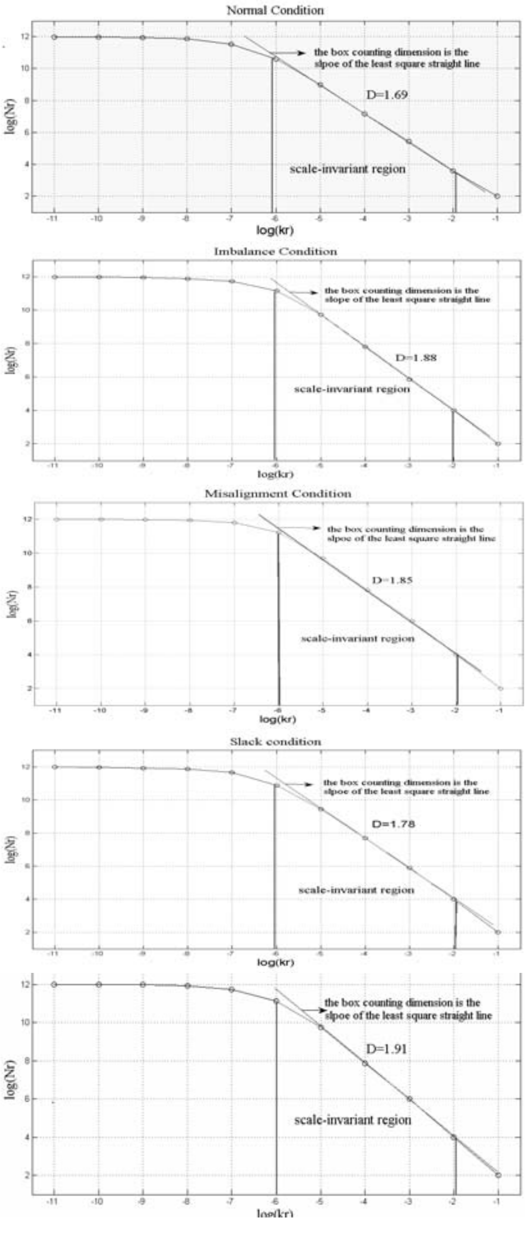

Fig. 7. The fractal logarithm curves of different fault conditions.

3.2 Extraction of fault characteristics

To extract the fault characteristics of each fault conditions, the coiflet 3-order wavelet packets transformation was used to divide the signals into five layers , i.e. N=5. The signal was divided into 32 frequency section, and each section's width was 31.25 Hz. The frequencies of fault conditions are mostly in base frequency, half base frequency, or multiple base frequencies. Because the base frequency F_b is 40 Hz, there are no two multiple frequencies existed simultaneously in the same frequency section. Extracting the box counting dimension of each frequency section accurately reflects the changing dimensions of different fault phenomena at different frequency sections. It was found that, under different fault conditions, there were obvious dimension changes in specific frequency sections, as shown in Table 1. The dimension change of Condition B occurred in the second frequency section, i.e. the frequency section where F_b exists. Condition C had most prominent dimension change in the fourth frequency section, i.e. $2Fb$. A dimensional characteristic of condition D was found in the eighth frequency section and Condition E in the eleventh frequency section. These results are consistent with the study of different fault conditions described in prior literature. Hence, this paper took the nine fre-

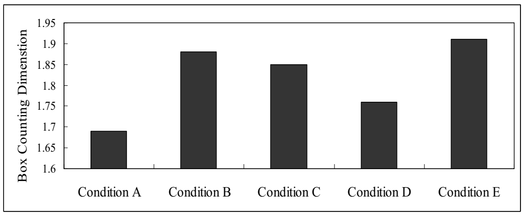

Fig. 8. Box counting dimension estimation results of different fault conditions.

Table 1. Box counting dimension characteristic vector.

Band serial number	Raw signal	S1 (5,0)	S2 (5,1)	S3 (5,2)	S4 (5,3)	S5 (5,4)	S6 (5,7)	S7 (5,8)	S8 (5,10)	S9 (5,11)
Frequency range	0~1000	0~ 31.25	31~ 62	62~ 93	93~ 125	125~ 156	218~ 250	250~ 281	312~ 343	343 ~375
Condition A	1.69	1.58	1.74	1.75	1.67	1.82	1.7	1.59	1.64	1.53
Condition B	1.88	1.68	1.89	1.73	1.82	1.82	1.69	1.7	1.62	1.65
Condition C	1.85	1.6	1.7	1.85	1.63	1.9	1.69	1.72	1.7	1.64
Condition D	1.76	1.36	1.8	1.64	1.78	1.78	1.68	1.80	1.68	1.66
Condition E	1.91	1.67	1.88	1.74	1.84	1.89	1.64	1.76	1.67	1.78

quency sections (i.e., section 0, 1, 2, 3, 4, 7, 8, 10 and 11) with obvious box counting dimension chan-ges. The box counting dimensions of the signals were set as characteristic vectors. The vectors were used as the inputs RBF neural network for learning and recog-nition of fault conditions.

3.3 RBF neural network learning and testing

In order to generate the input characteristics for the radial basis neural network, the box counting dimen-sion characteristic vector was normalized to be between 0 and 1. Each input vector has ten elements, it includes ten neurons in hidden layer, five nodes in output layer. The precision constant of training and learning was set to be 10^{-6}. Table 2 shows part of the training and learning results of the RBF neural network. From the table, it is shown that match very well.

Finally, the test samples were putted into the trained fault recognition system, and the results were shown in Table 3. Out of the 40 test samples of the five conditions, 39 were consistent with the recog-nition results. Only one test sample of condition was mistakenly identified as base looseness condition.

Table 2. The results of training and learning of RBF neural network.

RBF-NN input layer												
Training Sample	Box counting dimension characteristic vector(normalized)										Fault types	Training result
	Raw signal	S1 (6,0)	S2 (6,1)	S3 (6,2)	S4 (6,3)	S5 (6,4)	S6 (6,7)	S7 (6,8)	S8 (6,10)	S9 (6,11)		
1	0.62	0.4	0.7	0.71	0.57	0.84	0.62	0.42	0.51	0.42	A	A
2	0.84	0.44	0.86	0.54	0.72	0.72	0.46	0.48	0.32	0.38	B	B
3	0.82	0.5	0.62	0.82	0.53	0.88	0.61	0.65	0.62	0.55	C	C
4	0.79	0.24	0.84	0.62	0.81	0.81	0.68	0.71	0.68	0.65	D	D
5	0.9	0.28	0.82	0.46	0.71	0.84	0.20	0.51	0.28	0.74	E	E

Table 3. Accuracy of fault recognitions.

Test Sample	Number of input sample	Number of correct recognitions	Accuracy of fault recognitions (number of correct recognitions /number of input sample)
A	8	8	100%
B	8	8	100%
C	8	7	87.5%
D	8	8	100%
E	8	8	100%

4. Conclusion

This paper proposes a new fault diagnosis procedure which integrates wavelet packets fractal techniques and RBF neural networks. The wavelet packets can accurately decompose non-stationary signals into each perpendicular frequency bands without losing the characteristics of the raw signal. Fractal theory is used to extract the fault cha-racteristics of corresponding frequency bands and construct box counting dimension vectors to exhibit the fault characteristics. A RBF neural network is then used to perform the fault recognition of rotating machinery. The results of experimental investigation show the effectiveness and accuracy of using box counting dimension characteristic vectors as the input vector for the RBF network, whether in network training, learning or model recognition. Therefore, it is concluded that the combination of the wavelet packets fractal technique and radial neural network can provide an effective method to diagnose fault conditions of rotating machinery. Future works of this study will include the application of real rotating machines and the effectiveness of this diagnosis procedure for other types of fault conditions.

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